ELECTRIFICATION OF LOW-CONDUCTIVITY LIQUIDS IN LAMINAR FLOW THROUGH TUBES AND CAPILLARIES

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One of the possible mechanisms of the electrification of low-conductivity liquids (liquid hydrocarbons, mineral oils, etc.) flowing through metal tubes and capillaries is the oxidation–reduction reaction of impurity ions on channel walls [1-5]. This point of view was first considered in [2] on the basis of experimental data in [3], where it was shown that when a neutral liquid (N-heptane) flows through a grounded metal capillary it emerges positively charged, and the charging current from the end of the capillary is steady for a steady flow of liquid.

In contrast with [2, 4, 5] we assume here that the forward ion neutralization reaction rate is small in comparison with the inverse reaction rate.

1. Statement of the Problem. In contrast with conventional approach to the phenomenon of electrification of low-conductivity liquids based on the concept of the generation of a charging current as a result of the removal of charges from the diffusion part of the electric double layer by hydrodynamic flow [6], Gavis and Koszman [2] present a new approach which in terms of chemical kinetics consists in the following. They assume that at the channel entrance (Fig. 1) the neutral liquid contains positive B^{+z_1} and negative A^{-z_2} impurity ions, where z_1 and z_2 are the valences of the ions. During the motion of the liquid partial or complete neutralization of negative ions occurs on the tube wall ($r = R$) as a result of the reversible reaction

$$A^{-z_2} - me^- \underset{k_i}{\overset{k_f}{\rightleftarrows}} A^{-(z_2-m)}, \tag{1.1}$$

where k_f is the rate constant of the forward reaction and k_i of the inverse reaction; e^- is the electron, and m ($m \leq z_2$) is the number of electrons transferred from an A^{-z_2} ion to the electrode. For $m = z_2$, neutral molecules A are formed as a result of the reaction.

Since at the entrance Σ_0 ($x = 0$) the concentration of $A^{-(z_2-m)}$ ions (or A molecules) is zero, the forward reaction will proceed more rapidly than the inverse near the entrance. As a result an excess of positive ions is formed in the liquid, i.e., it is electrified.

The boundary-value problem describing the charging of the liquid in reaction (1.1) has the following form:

$$\operatorname{div} \varepsilon \mathbf{E} = 4\pi e[z_1 n_1 - z_2 n_2 - (z_2 - m) n_3], \ \mathbf{E} = -\nabla \psi; \tag{1.2}$$

$$\begin{gathered} \operatorname{div} \mathbf{i}_k = 0 \quad (k = 1, 2, 3), \ \mathbf{i}_1 = -D_1 \nabla n_1 + b_1 n_1 \mathbf{E} + n_1 \mathbf{v}, \ \mathbf{i}_2 = \\ = -D_2 \nabla n_2 - b_2 n_2 \mathbf{E} + n_2 \mathbf{v}, \ \mathbf{i}_3 = -D_3 \nabla n_3 - \gamma b_3 n_3 \mathbf{E} + n_3 \mathbf{v} (0 \leqslant r \leqslant R; \\ 0 \leqslant x \leqslant \infty); \end{gathered} \tag{1.3}$$

$$\Sigma_0 (x = 0): n_1 = n_1^0, \quad n_2 = n_2^0, \quad n_3 = 0, \quad z_1 n_1^0 = z_2 n_2^0; \tag{1.4}$$

$$S(r = R): \ \mathbf{i}_1 \mathbf{n} = 0, \ (\mathbf{i}_2 + \mathbf{i}_3) \mathbf{n} = 0, \ \mathbf{i}_2 \mathbf{n} = k_f n_2 - k_i n_3, \ \psi = \psi_0; \tag{1.5}$$

$$\text{as } x \to \infty \ \mathbf{i}_2 \mathbf{n} = 0, \ n_3/n_2 = k_f/k_i = K. \tag{1.6}$$

Here n_i, b_i, and D_i ($i = 1, 2, 3$) are, respectively, the partial concentrations, the mobilities, and the diffusion coefficients of the B^{+z_1}, A^{-z_2}, and $A^{-(z_2-m)}$ ions (or the A molecules; n_1^0 and n_2^0 are the constant concentrations of the B^{+z_1} and A^{-z_2} ions at the channel entrance; ε is the permittivity of the liquid; e is the charge of a proton; $\gamma = 1$ for $z_2 > m$, **n** is the outward normal to S.

Equations (1.2) and (1.3) assume that the space charge $e[z_1 n_1 - z_2 n_2 - (z_3 - m) n_3]$ formed in the electrification process, and the electric field induced by it, are so small that they do not affect the velocity distribution of the

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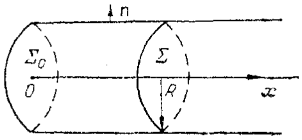

Fig. 1

liquid, i.e., $\mathbf{v} = v_0(0, 0, v(r))$ is a given function of coordinates, where v_0 is the velocity on the channel axis. The last of Eqs. (1.4) follows from the neutrality of the liquid at the entrance.

The first of Eqs. (1.5) states that there is no current of positive charges to the earth. The second equation represents the equality of the fluxes of A^{-z_2} and $A^{-(z_2-m)}$ ions at the channel surface S, which is a consequence of the fact that each $A^{-(z_2-m)}$ (A^{-z_2}) ion can be formed on S only as a result of the forward (inverse) reaction (1.1), and that they are not adsorbed on S. The third condition follows from the fact that the difference between the number $k_f n_2$ of A^{-z_2} ions neutralized and the number $k_i n_3$ of $A^{-(z_2-m)}$ ions regenerated on S is equal to the influx of A^{-z_2} (or $A^{-(z_2-m)}$) ions. Here it is assumed that (1.1) is a single-stage reaction [7]. The potential $\psi = \psi_0 = \text{const}$ on S, since the channel wall is grounded.

Conditions (1.6) express the equilibrium of reaction (1.1) at a sufficiently large distance from the initial cross section Σ_0.

The charging current through an arbitrary cross section Σ ($x = \text{const}$) is

$$I = e\int_{\Sigma} [z_1 \mathbf{i}_1 - z_2 \mathbf{i}_2 - (z_2 - m)\, \mathbf{i}_3]\, \mathbf{e}_x d\Sigma. \tag{1.7}$$

2. Method of Solution. Just as in [2, 5], we make the following assumptions to simplify further study of the problem. First, we assume a sufficiently large average flow velocity of the liquid so that all quantities change much more slowly along the channel axis then in the radial direction. Hence, derivatives with respect to x can be omitted in the Laplacian Δ. Second, we assume that the fraction of converted A^{-z_2} ions is small in comparison with the unconverted ions in any cross section Σ. Under this assumption the equilibrium constant K of reaction (1.1) is small enough to permit the use of perturbation theory. Finally, we assume the equality of diffusion coefficients $D_i = D$ and mobilities $b_i = b$ ($i = 1, 2, 3$).

In accordance with the perturbation method, we seek the solution in the form of series

$$\begin{gathered} n_i = n_i^0 + n_i^{(1)} + n_i^{(2)} + \ldots,\ n_3 = n_3^{(1)} + n_3^{(2)} + \ldots, \\ \psi = \psi_0 + \psi^{(1)} + \psi^{(2)} + \ldots \quad (i = 1, 2). \end{gathered} \tag{2.1}$$

After substituting (2.1) into (1.2)-(1.6), linearizing, and transforming to dimensionless variables by the formulas

$$\begin{gathered} q_i = n_i^{(1)}/n^0 \ (i = 1, 2, 3), \quad n^0 = z_1 n_1^0 + z_2 n_2^0, \\ \varphi = \psi^{(1)}\varepsilon/4\pi e n^0, r' = r/R,\ x' = x/R, \end{gathered}$$

we obtain

$$\begin{gathered} -L\varphi = q,\ Lq_3 - \varkappa v(r)\partial q_3/\partial x = 0, \\ Lq - \delta^2 q - \varkappa v(r)\partial q/\partial x = 0; \end{gathered} \tag{2.2}$$

$$\text{at } x = 0 \ q = q_3 = 0; \tag{2.3}$$

$$\text{at } r = 1 \ \frac{\partial q}{\partial r} + \delta^2 \frac{\partial \varphi}{\partial r} = m\left(K_f \frac{\delta_2^2}{\delta^2} - K_i q_3\right) = m\frac{\partial q_3}{\partial r}. \tag{2.4}$$

Here

$$\begin{gathered} q = z_1 q_1 - z_2 q_2 - (z_2 - m)\, q_3; \quad L \equiv \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}; \\ \delta_2^2 = R^2/D\tau_2 \ \left(\tau_2 = \varepsilon/4\pi e b n_2^0\right); \quad \delta^2 = R^2/D\tau \ (\tau = \varepsilon/4\pi e b n^0); \\ \varkappa = v_0 R/D;\ K_f = k_f R/D;\ K_i = k_i R/D \end{gathered}$$

and primes on the dimensionless coordinates have been omitted.

We assume a Poiseuille velocity profile $v(r) = 1 - r^2$. This approximation is valid when the characteristic length for the development of a parabolic profile $l \sim 0.16 v_0 R^2/\nu$ [8], where ν is the kinematic viscosity, is appreciably shorter than the characteristic channel length L over which the charging of the liquid takes place.

We solve problem (2.2)-(2.4) by taking Laplace transforms.

$$q_* = \int_0^\infty q(r, x)\, e^{-px} dx$$

Then we obtain the following problem for the transforms:

$$-L\varphi_* = q_*, \quad Lq_* - [\delta^2 + \beta^2(1 - r^2)]\, q_* = 0, \quad Lq_{3*} - \beta^2(1 - r^2)\, q_{3*} = 0, \quad \beta^2 = \varkappa p; \tag{2.5}$$

$$\frac{dq_*}{dr} + \delta^2 \frac{d\varphi_*}{dr} = m\left(K_f \frac{\delta_2^2}{\delta^2 p} - K_i q_{3*}\right) = m \frac{dq_{3*}}{dr} \quad (r = 1), \tag{2.6}$$

which we solve by making the substitutions

$$q_* = A_1 e^{-z/2} \Phi(\alpha_1, 1, z), \quad q_{3*} = A_2 e^{-z/2} \Phi(\alpha_2, 1, z),$$
$$\alpha_1 = 1/2 + (\delta^2 + \beta^2)/4i\beta, \quad \alpha_2 = 1/2 + \beta/4i, \quad z = i\beta r^2,$$

where $\Phi(\alpha, 1, z)$ is the confluent hypergeometric function [9]. After determining the constants A_1 and A_2 from (2.6), and taking the inverse Laplace transforms, we obtain

$$q = \frac{m\delta_2^2 K_f}{\delta^2} \frac{1}{2\pi i} \int_{p_0 - i\infty}^{p_0 + i\infty} \frac{i\beta H(\beta)\, e^{\frac{i\beta(1-r^2)}{2}} \Phi\left(\alpha_1, 1, i\beta r^2\right)}{pH_1(\delta, \beta)\, H_2(K_i, \beta)} e^{px} dp \; (\operatorname{Re} p_0 > 0), \tag{2.7}$$

$$H(\beta) = 2\alpha_2 \Phi(\alpha_2 + 1, 2, i\beta) - \Phi(\alpha_2, 1, i\beta),$$

$$H_1(\delta, \beta) = i\beta \left[2\alpha_1 \Phi(\alpha_1 + 1, 2, i\beta) - \Phi(\alpha_1, 1, i\beta)\right] - \frac{\delta^2}{2} \int_0^1 e^{\frac{i\beta(1-\xi)}{2}} \Phi(\alpha_1, 1, i\beta\xi)\, d\xi,$$

$$H_2(K_i, \beta) = K_i \Phi(\alpha_2, 1, i\beta) + i\beta H(\beta).$$

Using the theorem of residues [9], we can write (2.7) in the form

$$q = \frac{m\delta_2^2 K}{4\delta^2} \left(\frac{I_0(\delta r)}{\int_0^1 (1 - r^2)\, I_0(\delta r)\, r dr} - \sum_{k=1}^{2} \sum_{n=1}^{\infty} a_{kn} e^{\frac{p_{kn}}{2}(r^2 - 1)} \Phi(\alpha_{kn}, 1, -p_{kn} r^2) e^{-\frac{p_{kn}^2}{\varkappa} x} \right), \tag{2.8}$$

where $\alpha_{1n} = 1/2 - (\delta^2 - p_{1n}^2)/4p_{1n}$; $\alpha_{2n} = 1/2 + p_{2n}/4$; p_{1n} and p_{2n} are the positive simple roots, numbered in ascending order, of the following equations:

$$h_1(p_{1n}) \equiv H_1(\delta, ip_{1n}) = p_{1n}[-2\alpha_{1n}\Phi(\alpha_{1n} + 1, 2, -p_{1n}) + \Phi(\alpha_{1n}, 1, -p_{1n})] - \frac{\delta^2}{2} \int_0^1 e^{\frac{p_{1n}}{2}(\xi - 1)} \Phi(\alpha_{1n}, 1, -p_{1n}\xi)\, d\xi = 0; \tag{2.9}$$

$$h_2(p_{2n}) \equiv H_2(K_i, ip_{2n}) = K_i \Phi(\alpha_{1n}, 1, -p_{2n}) - p_{2n}[2\alpha_{2n}\Phi(\alpha_{2n} + 1, 2, -p_{2n}) - \Phi(\alpha_{2n}, 1, -p_{2n})] = 0. \tag{2.10}$$

The coefficients a_{1n} and a_{2n} are found in the form

$$a_{1n} = \frac{8K_i H(ip_{1n})}{h_1'(p_{1n})\, H_2(K_i, ip_{1n})}, \quad a_{2n} = \frac{8K_i H(ip_{2n})}{H_1(\delta, ip_{2n})\, h_2'(p_{2n})}, \tag{2.11}$$

where primes denote derivatives with respect to p_{1n} and p_{2n} respectively.

Using (1.7), and taking account of (2.8), we find the charging current

$$I = I_\infty \left[1 - \sum_{n=1}^{\infty} a_n \exp\left(-\frac{p_{1n}^2 D}{v_0 R^2} x\right) - \sum_{n=1}^{\infty} b_n \exp\left(-\frac{p_{2n}^2 D}{v_0 R^2} x\right)\right]; \tag{2.12}$$

$$I_\infty = \frac{1}{2} m\pi e n_2^0 v_0 R^2 K, \quad a_n = a_{1n} c_{1n}, \quad b_n = a_{2n} c_{2n}, \tag{2.13}$$

$$c_{kn} = \frac{1}{2} \int_0^1 (1 - \xi) \exp\left(\frac{p_{kn}}{2}(\xi - 1)\right) \Phi(\alpha_{kn}, 1, -p_{kn}\xi)\, d\xi$$
$$(k = 1, 2, \; n = 1, 2, 3, \ldots).$$

3. Results of Numerical Calculations. Equations (2.9) and (2.10) show that the p_{1n} depend only on the parameter δ^2, and the p_{2n} on K_i. The results of numerical calculations of these dependences plotted in Figs. 2 and 3 show that as $\delta \to 0$ the roots p_{1n} approach constant values ($p_{11} \to 5.0$, $p_{12} \to 9.1$, $p_{13} \to 12.5$), whereas as δ

TABLE 1

δ^2	1	30	50	70	100
b_1	0,835	0,836	0,831	0,822	0,808
b_2	0.97	0,095	0,095	0,095	0,094
b_3	0,014	0,014	0,014	0,014	0,013
a_1	$0.28\cdot10^{-5}$	$0,28\cdot10^{-4}$	$0,15\cdot10^{-3}$	$-0,0013$	$0,5\cdot10^{-6}$

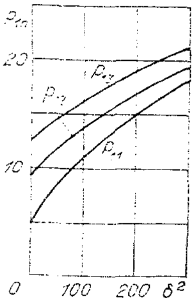

Fig. 2

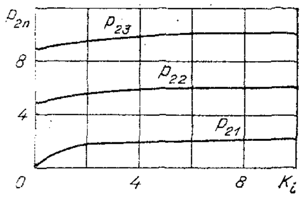

Fig. 3

increases, they increase; for $\delta > 10$ we have $p_{11} \sim \delta$ (for $\delta^2 = 1$, 10, 50, and 100, $p_{11} = 5.16$, 5.97, 8.72, and 11.2, respectively). The roots p_{2n} also increase monotonically with increasing K_i, but, in contrast with the previous case, approach constant values as $K_i \to \infty$. Thus, for $K_i \geq 40$, the p_{2n} ($n = 1$, 2, 3) change by a fraction of a percent, and their values for $K_i = 10$ and $K_i = 40$ differ by 3-4% (for $K_i = 10$, $p_{21} = 2.51$, $p_{22} = 6.32$, $p_{23} = 10.12$; for $K_i = 40$, $p_{21} = 2.65$, $p_{22} = 6.59$, $p_{23} = 10.55$).

The coefficients a_n and b_n in series (2.12) found by using these numerical values in Eqs. (2.11) and (2.13) are listed in Table 1. Calculations showed that the b_n are positive and decrease rapidly with increasing n (for $n = 1$, 2, 3 approximately by an order of magnitude when n increases by unity (Table 1)); the coefficient b_1, which makes the main contribution to the series sum, decreases slowly and monotonically with increasing δ (the values of b_1 for $\delta^2 = 1$ and $\delta^2 = 100$ differ by 2%). The coefficients a_n may change sign, but their absolute values are small for $\delta^2 \leq 100$: $|a_n| \ll b_n$ ($n = 1$, 2, 3). Calculations also showed that for $n = 1, 2, 3$ the absolute values of the a_n can be of the same order of magnitude.

Thus, for $\delta^2 \leq 100$ and $K_i \geq 40$, the expression for the charging current, to within terms $0(10^{-3})$, can be written in the form

$$I = I_\infty \left[1 - \sum_{n=1}^{3} b_n \exp\left(-\frac{\pi p_{2n}^2 D}{2Q} x\right)\right] \quad (I_\infty = emn_2^0 QK),$$

where $Q = (\pi/2) v_0 R^2$ is the liquid flow rate; $p_{21}^2 = 6.7$; $p_{22}^2 = 43.5$; $p_{23}^2 = 111$.

Table 1 shows that as the parameter δ is increased, the coefficients b_n decrease, and the $|a_n|$ increase. Thus, as $\delta \to \infty$ one might assume that the series with the coefficients a_n will play the main role in Eq. (2.12) for the charging current. We could not test this assumption by numerical computer calculations, however, because of the poor convergence of the series for the confluent hypergeometric function and the presence of the rapidly varying exponential factor in the integrands in (2.9) and (2.13). On the other hand, for very large δ the condition for the asymptotic convergence of (2.1), which can be written in the form $\delta K \ll 1$, is violated, and therefore the asymptotic solution obtained from (2.7) as $\delta \to \infty$ has no meaning.

Thus, the analysis given is valid for sufficiently small equilibrium constants $K \ll 1$, and K_i, and $\delta^2 \ll K^{-2}$.

Analysis of the Solution. Equation (3.1) shows that the charging current increases monotonically with increasing flow rate Q, and for small values of Q ($Q \ll 2\pi p_{23}^2 Dx$) it varies linearly ($I = emn_2^0 KQ$). In the limit as $Q \to \infty$ the current approaches saturation:

$$I \to I_0 = \frac{\pi}{2}\left(b_1 p_{21}^2 + b_2 p_{22}^2 + b_3 p_{23}^2\right) emn_2^0 DKx.$$

We note that this dependence of I on the average velocity of laminar flow in a capillary is observed in experiments [3] (for turbulent flow $I \sim v_0^{15/8}$ at low velocities, and $I \sim v_0^{7/8}$ at high velocities [3]). The dependence of I on the ion concentration at the capillary entrance is generally treated as a dependence on conductivity. In this sense (3.1) gives a linear law, which is observed in experiments at sufficiently low conductivity [3]. However, in the light of modern concepts of the conductivity of liquid dielectrics [10, 11], a steady current is not ensured by impurity ions, but by injection (or regeneration [10]) processes at the electrode–liquid contact, which are determined by the physical and chemical properties of the electrode, the liquid, and the impurity component. The last, according to data on electroconvective flows, must be neutral [11]. Thus, all one can say is that the conductivity of a neutral liquid is proportional to the ion concentration in it. Therefore, to explain the experimentally observed decrease in the charging current with an increase in conductivity for sufficiently high values of them, it is necessary to consider the interaction of the electrode surface not only with the impurity ions, but also with the neutral impurity component. Thus, for a sufficiently high concentration c_X of impurity X (i.e., high conductivity), their adsorption on the capillary surface can lead to a slowing down of the rate of neutralization of negative ions, i.e., to a decrease in the charging current. Analytically this indicates that for sufficiently large c_X the limiting charging current I_∞ becomes a monotonically decreasing function of c_X. These arguments are confirmed experimentally in [3] where it was shown that as long as $\sigma < \sigma_0$ the limiting current I_∞ remains constant as σ is increased, and begins to decrease only for sufficiently large $\sigma > \sigma_0$. From a quantitative point of view this can be described in the following way. If we assume that on that part of the surface where X was adsorbed the neutralization of negative ions (1.1) does not occur, the third condition in (1.5) takes the form

$$\mathbf{i}_2\mathbf{n} = (1 - \lambda)(k_f n_2 - k_i n_3),$$

where λ is the area occupied by the adsorbate X per unit area of the capillary surface. For example, if adsorption is described by the Langmuir isotherm, $\lambda = \beta c_X/(1 + \beta c_X)$, where β is a physical constant depending on the electrode material and the kind of impurity X. In this case the charging current will also be determined by (2.12), where $I_\infty = 1/2\, m\pi\, (1 - \lambda) e n_2^0 v_0 R^2 K$. Since c_X and λ are proportional to the conductivity of the liquid, as $\sigma \to 0$, $\lambda \to 0$, and as $\sigma \to \infty$, $\lambda \to 1$. Therefore, for small conductivities the charging current varies linearly with σ, while for large conductivities it decreases with increasing σ.

The modern point of view on the conductivity of liquid dielectrics makes it possible to propose a fundamentally different electrification mechanism. Thus, if the electrode atoms (molecules) M manifest positive or negative electronegativity [12] with respect to the liquid molecules A (impurity X), positive or negative ions may be formed on the electrode surface which migrate into the depths of the liquid by convection or diffusion and electrify it. This mechanism of ion formation on the electrode surface is based on the concept of the conductivity of liquid dielectrics in [13].

It follows from (3.1) that for large enough x ($x \gg 2Q/\pi p_{21}^2 D$) the charging current is independent of the length of the capillary and is equal to $I = I_\infty = emn_2^0 QK$. The characteristic length L (electrification length) over which the limiting current I_∞ can be reached is estimated as $L = v_0 R^2/p_{21}^2$. For typical values $D = 10^{-5}$ cm²/sec under the conditions of the experiments with heptane [3] ($v_0 = 100$ cm/sec, $R = 0.016$ cm), we have $L = 400$ cm. Such large values of the electrification length can be accounted for as follows. The electrification process ceases when reaction (1.1) comes to equilibrium. This equilibrium is reached when the concentration n_3 of $A^{-(z_2 - m)}$ ions becomes uniform over the whole volume and equal to $n_3 = Kn_2^0$. Within the framework of the approximation considered, this process is determined by diffusion only (2.2). For high liquid velocities ($v_0 \geq 100$ cm/sec) the convective flux is appreciably larger than the diffusion flux, which also leads to larger values of L. The smallest length L_m for which significant electrification ($I \simeq 0.1\, I_\infty$) is possible is estimated as $L_m = v_0 R^2/p_{23} D$. For the values of v_0, R, and D given above we have $L_m = 25$ cm. We note that in experiments [3] the length of capillaries in which appreciable electrification was observed was 4-40 cm, which agrees with the above estimate.

Thus, for low conductivity there is qualitative agreement of (3.1) with the experimental data of [3]. For a quantitative test of (3.1) and qualitative agreement for high conductivities, further experimental research is necessary to determine the components of the reaction (1.1), to measure the reaction rate constants (equilibrium constants K), and to study the role of neutral impurities.

LITERATURE CITED

1. V. V. Zakharchenko et al., Electrification of Liquids and Its Prevention [in Russian], Khimiya, Moscow (1975).
2. J. Gavis and I. Koszman, "Development of charge in low-conductivity liquids flowing past surfaces: a theory of phenomenon in tubes," J. Coll. Sci., 16, 375-391 (1961).
3. I. Koszman and J. Gavis, "Development of charge in low-conductivity liquids flowing past surfaces. Experimental verification and application of the theory developed for tube flow," Chem. Eng. Sci., 17, 1023-1040 (1962).
4. V. V. Gogosov, E. I. Nikiforovich, and V. V. Tolmachev, "Electrization of a low-conductivity liquid flowing in a metal tube," Magn. Gidrodin., No. 2, 59-62 (1979).

5. V. N. Pribylov and L. T. Chernyi, "Electrification of dielectric liquids flowing in tubes," Izv. Akad. Nauk SSSR, Mekh. Zhid. Gaza No. 6 (1979).
6. L. B. Leb, Static Electrification [in Russian], Gostekhizdat, Moscow–Leningrad (1963).
7. Ya. I. Gerasimov et al., Course in Physical Chemistry [in Russian], Vol. 2, Khimiya, Moscow (1973).
8. I. A. Slezkin, Dynamics of a Viscous Incompressible Fluid [in Russian], Gostekhizdat, Moscow (1955).
9. D. S. Kuznetsov, Special Functions [in Russian], Vysshaya Shkola Moscow (1962).
10. Yu. K. Kopylov, "Fundamental laws of the electrical conductivity of high-resistance organic liquids for an activation character of contact processes," Organic Semiconducting Liquids, Vol. 27 [in Russian], Trudy Dnepropetrovosk Inst. (1974).
11. A. I. Zhakin, "Some problems of the electrohydrodynamic stability and the electroconvection of incompressible fluids," Author's Abstract of Candidate's Dissertation, Kharkov State Univ. (1980).
12. N. L. Glinka, General Chemistry [Russian translation], Khimiya, Leningrad (1972).
13. L. S. Kazatskaya and I. M. Solodovnichenko, "On the role of electroinduction effects of molecules in the production of charge carriers in an organic liquid," Elektron. Obrab. Mater., No. 2 (1979).

INITIAL ASYMPTOTE TO THE SOLUTION OF THE PROBLEM OF DROPLET INCIDENCE ON A PLANE

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The initial stage of collision of a spherical droplet on a solid plane is considered. It is assumed that the droplet liquid is ideal and incompressible, and that surface tension and external mass forces are absent.

This problem is closely related to that of entry of a blunt body into a liquid, which was first considered in [1]. The method for calculation of the resistive forces, developed in [1], is based on the assumption that the velocity distribution on the free surface at each moment is the same as that obtained directly after collision of a floating plate of the same dimensions.

These problems have the following unique features: 1) the flow region Ω_t is unknown; 2) the contact line between free liquid surface and the solid must be determined at the boundary of the flow region; 3) singularities may appear in the solution on this line.

A new approach to problems of this kind is the introduction of Lagrangian coordinates [2, 3], in which the flow region is fixed.

1. At time t = 0 a liquid sphere of radius a is tangent upon a solid plane, which moves along the z axis at velocity v. We must find the liquid motion which then occurs. In the space formed by Lagrangian Cartesian coordinates ξ, η, ζ the region occupied by the liquid is known, being a sphere of radius a with center at the origin. We denote this region by Ω_0. The variables x, y, z denote the corresponding Euler coordinates, Γ is the free surface of the liquid, and Σ is the contact spot between droplet and solid plane. The Euler equations, written in Lagrangian coordinates, have the form [3]

$$M_0^* \mathbf{x}_{tt} + \frac{1}{\gamma} \nabla_\xi p = 0, \quad \det M_0 = 1 \text{ in } \Omega_0 \tag{1.1}$$

with boundary conditions $p|_\Gamma = 0$, $z_t|_\Sigma = v$ and initial conditions $\mathbf{x}|_{t=0} = \xi$, $\mathbf{x}_t|_{t=0}$, where $\mathbf{x} = (x, y, z)$; $\boldsymbol{\xi} = (\xi, \eta, \zeta)$; $M_0 = \partial(\mathbf{x})/\partial(\boldsymbol{\xi})$; M_0^* is the matrix conjugate to M_0 and p is the pressure. The problem is a complex one because of its nonlinearity and the existence of the unknown line on the sphere boundary $\partial\Omega_0$, dividing Γ and Σ.

2. We will linearize Eq. (1.1) for the initial rest state, keeping terms of zeroth- and first-order smallness in displacement. For the linearized problem we can introduce a displacement potential $\Phi = \Phi(\xi, \eta, \zeta, t)$, which in view of the continuity equation, will be a function harmonic in Ω_0. From the momentum equation follows that

$$p = -\gamma \Phi_{tt} \tag{2.1}$$

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